Grade 3 Mathematics, Quarter 2, Unit 2.1

Understanding Properties of Division

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Recognize that division is the whole number of objects divided by the equal shares.
- Find the unknown whole number in a division equation.
- Apply properties of operations as strategies to divide.
- Recognize division as the inverse relationship of multiplication.
- Fluently multiply and divide within 100.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain relationships between equations.
- Check answers to a problem using a different method.

Reason abstractly and quantitatively.

- Know and flexibly use different properties of operations and objects.
- Flow between contextual and non-contextual situations.

Attend to precision.

- Calculate accurately by rechecking for precision.

Essential questions

- What does each number represent in a division problem?
- How do you determine the unknown in a division equation?
- How are multiplication and division related?
- How could you use properties of multiplication to help you solve a division equation?
- What is your strategy for mental multiplication?
- What is your strategy for mental division?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

[2] Students need not use formal terms for these properties.

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and

relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students used addition to find the total number of objects in rectangular arrays with up to five rows and five columns. They wrote equations to express the total as a sum of equal addends. They skip counted by 2s and used repeated addition to find the total number of objects.

Current Learning

In grade 3, students develop an understanding of the meaning of division of whole numbers through activities and problems involving equal size groups, arrays, and area models; division is finding an unknown factor. Students understand division as a whole number partitioned into equal groups. They also apply properties of multiplication to help solve division equations. This is all at the developmental level. By the end of third grade, students are expected to fluently multiply and divide within 100.

Routines: Students will continue to use estimation strategies to check for reasonableness of their results in the context of division.

Future Learning

In grade 4, students will interpret a multiplication equation as a comparison. They will represent verbal statements of multiplicative comparisons as multiplication equations. They will use all four operations to solve word problems. They will illustrate and explain the calculation by using equations, the area model and rectangular arrays. They also will find whole number quotients and remainders with up to four-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

Additional Findings

Principles and Standards for School Mathematics states, "In grades 3–5, students should focus on the meanings of, and relationship between, multiplication, and division. It is important that students understand what each number in a multiplication or division expression represents. For example, in multiplication, unlike addition, the factors in the problem can refer to different units. If students are solving the problem 24 × 9 to find out how many legs there are on 29 cats, 29 is the number of cats (number of groups), 4 is the number of legs on each cat (or the number of items in each group), and 116 is the total number of legs on all the cats. Modeling multiplication problems with pictures, diagrams, or concrete materials helps students learn what the factors and their product represent in various contexts" (p. 151).

It also states, "students should consider different types of problems that can be solved using multiplication and division. For example if there are 112 people traveling by bus and each bus can hold 28 people, how many buses are needed? In a different problem, students might know the number of groups and need to find how many items there are in each group. For example if 112 people divide themselves evenly among four 4 buses, how many people are on each bus? Students need to see each type of problems as division situations, and should know the units of the result" (p. 151).

Progressions for the Common Core State Standards in Mathematics states, "because an unknown factor (a division) can be found from the related multiplication, the emphasis at the end of the year is on knowing from memory all products of two-digit numbers. As should be clear from the foregoing, this isn't a matter of instilling facts divorced from their meaning, but rather the outcome of a carefully designed learning process that heavily invokes the interplay of practice and reasoning.

Grade 3 Mathematics, Quarter 2, Unit 2.2

Applying Multiplication and Division Strategies to Problem Solving

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Use multiplication and division within 100 to solve word problems in situations involving equal groups and arrays to find the unknown product, group size unknown, and number of groups unknown.
- Use multiplication and division within 100 to solve word problems in situations involving measurement quantities.
- Solve two-step word problems using the four operations using equations with a letter standing for the unknown quantity.
- Multiply one-digit whole numbers by multiples of 10 (10-90) using strategies based on place value and properties of operations.
- Use estimation strategies and mental computation to assess the reasonableness of an answer.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain to themselves the meaning of the problem.
- Check the answer to problems using a different method.

Model with mathematics.

- To solve problems in everyday life through the use of algorithms.

Essential questions

- When given a word problem, how do you know which information is needed to solve it?
- How do you decide which operation(s) will be best to solve a problem?
- How do you write an equation using a letter for an unknown quantity to help solve a word problem?
- Explain how estimation and mental computation can help you to check if your answer makes sense.
- How can understanding place value help you multiply multiples of 10?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Glossary, Table 2.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]

[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

[4] A range of algorithms may be used.

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 2, students used addition to find the total number of objects in rectangular arrays with up to 5 rows and 5 columns. They wrote equations to express the total as a sum of equal addends. They skip count by 2's and used repeated addition to find the total number of objects. Students use addition and subtraction within 100 to solve one and two-step word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions.

Current Learning

In grade 3, students solve two-step word problems using the four operations. They represent these problems using equations with a letter standing for the unknown quantity. They also assess the reasonableness of answers using mental computation and estimation strategies including rounding. At this level, students are working with whole numbers and the answers are whole numbers. The operations of addition and subtraction (within a 1000) are at the reinforcement level. Multiplication and division problems (within 100) are at the developmental level. It is important to reference table 2 in the glossary of the CCSS. Students multiply one-digit whole numbers by multiples of 10 in the range of 10-90 using strategies of place value and properties of operations. This is at the developmental level.

Routines: Students will continue to use estimation strategies to check for reasonableness of their results within problem solving.

Students will work to increase fluency with multiplication facts

Future Learning

In grade 4, students will interpret a multiplication equation as a comparison. They will represent verbal statements of multiplicative comparisons as multiplication equations. They will solve multi-step word problems posed with whole numbers and having whole number answers using all four operations. They also will solve problems in which remainders must be interpreted. They will continue to represent these problems using equations with a letter for the unknown quantity. They will continue to assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Student will recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

Additional Findings

Progressions for the Common Core State Standards in Mathematics (draft) it states, "because an unknown factor (a division) can be found from the related multiplication, the emphasis at the end of the year is on knowing from memory all products of two-digit numbers. As should be clear from the foregoing, this isn't a matter of instilling facts divorced from their meaning, but rather the outcome of a carefully designed learning process that heavily invokes the interplay of practice and reasoning.

The Common Core State Standards for Mathematics states that, "Students use properties of operations to calculate products of whole numbers using increasingly sophisticated strategies based on these properties to solve multiplication and division problems using single digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division."

PARCC Model Content Frameworks Mathematics Grades 3-11 states that "Finding single-digit products and related quotients is a required fluency fro grade 3. Reaching fluency will take much of the year for many students. These skills and the understandings that support them are crucial; students will rely on them for years to come as they learn to multiply and divide with multi-digit whole numbers and to add, subtract, multiply and divide with fractions. After multiplication and division situations have been established, reasoning about patterns and products can help students remember particular products and quotients. Practice- and if necessary, extra support- should continue all year for those who need it to attain fluency."

Grade 3 Mathematics, Quarter 2, Unit 2.3

Measuring Area

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Partition shapes into parts with equal areas understanding that the equal parts of a fraction can be referred to having the same area.
- Describe the area of each part of a whole that was partitioned, as a fraction.
- Recognize that square units are used to measure area.
- Identify that a plane figure covered with unit squares without gaps or overlaps has an area of the number of covered unit squares.
- Measure the area of a plane figure by counting the square units that are in the plane figure.
- Label the area of a figure using the correct measurement unit. (square cm, square m, square in, square ft, and improvised units).

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the relationships between area and fractional parts of a whole.
- Monitor and evaluate their progress.

Use appropriate tools strategically**.**

- Using estimations to judge reasonableness of solutions.

Attend to precision.

- Label for clarification.
- Calculate accurately by rechecking for precision.
- Define units of measure consistently and appropriately.

Essential questions

- How can you partition a given shape into parts with equal areas?
- What fraction name would you give to express the area of one part of the whole?
- Why is it important that the unit squares do not leave gaps or overlap on a plane figure when solving for area?
- What is the area of a given plane figure?
- Why is it important to label square units when solving for area?
- Why is important to use the correct measurement unit?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 3.G

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape*.

Measurement and Data 3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students in grade 2 partitioned a rectangle into rows and columns of same size squares and counted them. They also partitioned circles and rectangles into 2, 3, or 4 equal shares. They measured the length of an object by choosing and using appropriate tools such as rulers, yardsticks, meter sticks and measuring tapes. Students measured an object twice using different length units such as: *in*, *ft*, *cm*, and *m* to notice the difference in length between the units.

Current Learning

In grade 3, students partition a whole into equal parts and recognize the parts have equal areas. This is at the developmental level. Students recognize that a square with a side length of one unit, called a unit square, is said to have one square unit of area. They recognize that a plane figure covered with a certain number of unit squares, that do not overlap or have gaps, has an area of that many square units. They measure area by counting the unit squares and label it correctly. (square cm, square m, square in, square ft, and improvise units.) This is all at the developmental level.

Routines: Students will continue to practice multiplication and division facts to achieve fluency.

Future Learning

Students in grade 4 will know relative sizes of measurement units within one system of units including km, m, cm, ft, yd, and in. They will record measurement equivalents in a two column table. Students will apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics states, "using manipulatives can facilitate the construction of sound representations of geometric concepts, but they must be used thoughtfully. If not, students may merely learn rote manipulation"(p. 155). The book further states that "imprecise language plagues students' work in geometry. Instruction should carefully draw distinctions between common usage and mathematical usage" (p. 154). And, "students instructed in geometry with computers often score significantly higher than those having just classroom instruction, from the elementary years to high school" (p. 156).

Principles and Standards for School Mathematics states, "the study of geometry in grades 3–5 requires thinking *and* doing. As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop. At the same time they are learning to reason, and to make, test, and justify conjectures about these relationships. This exploration requires access to a variety of tools, such as, graph paper, rulers, pattern blocks, geoboards, and geometric solids, and is greatly enhanced by electronic tools that support exploration, such as dynamic geometry software" (p. 165)

Progressions for the Common Core State Standards in Mathematics (draft) states that: "Initially students can use an intuitive notion of congruence (same size and same shape) to explain why the parts are equal…. Students come to understand a more precise meaning for "equal parts" as "parts with equal measurements." For example, when a ruler is partitioned into halves or quarters of an inch they see that each subdivision has the same length. In area models, they reason about the area of the shaded region to decide what fraction of the whole it represents.